Nonlinear radiation pressure and stochasticity in ultraintense laser fields

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The drift acceleration due to radiation reaction for a single electron in an ultraintense plane wave ($a = eE/mc\omega \sim 1$) of arbitrary wave form and polarization is calculated and shown to be proportional to a^3 in the high-a limit. The resulting average drift acceleration is independent of polarization, even though the average radiated power is polarization-dependent. The cyclotron motion of an electron in a constant magnetic field and an ultraintense plane wave is numerically found to be quasiperiodic even in the high-a limit if the magnetic field is not too strong, as suggested by previous analytical work. A strong magnetic field causes highly chaotic electron motion and the boundary of the highly chaotic region of parameter space is determined numerically and shown to agree with analytical predictions. [S1063-651X(99)06102-4]

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It has been known for many years that qualitative changes occur in the behavior of an electron moving in an electromagnetic plane wave when the dimensionless strength $a = eE/mc\omega$ is of order unity. Recent advances in laser pulse compression and amplification [1] have made it possible to attain such ultraintense waves in the laboratory and led to new investigations of their properties. One important effect is that an electron moving in an ultraintense electromagnetic plane wave and also subjected to slowly varying ''background'' electric or magnetic fields behaves approximately like a particle of enhanced mass $m\gamma_0 = m\sqrt{1+e^2\langle A_\mu A^\mu\rangle/m^2c^4}$ drifting in the background fields. Here A is the vector potential and $\gamma_0=\sqrt{1+a^2/2}$ for a linearly polarized monochromatic wave. The fast motion in the wave can be rigorously averaged over if the background fields are sufficiently weak and the plane wave is not so strong that pair creation effects become significant. Except for some brief remarks on collective behavior at the end, this paper considers the behavior of a single electron.

This enhanced-mass approximation has a long history going back at least to the work of Volkov [2]. In a previous paper its validity was shown for sufficiently weak and slowly varying background fields (note that a slow change in the wave envelope can be treated for this purpose as a ''background field,'' causing the standard longitudinal ponderomotive force on an oscillating electron) [3]. In the course of that derivation, general equations were found for the response of the electron guiding center (i.e., the center of the electron's fast motion in a harmonic wave) to an impulse. The first part of this paper uses the guiding-center equations to calculate the drift acceleration of an electron in a plane wave due to the radiation damping and reaction forces. This results in a generalization for strong classical waves of the acceleration due to radiation damping. To our knowledge the correct average accleration of an electron in a strong wave due to radiation reaction has not previously been obtained. In a strong wave, the variation of the electron effective mass $m\gamma$ over the wave orbit must be properly accounted for to obtain the correct drift acceleration. The average drift acceleration is the quantity of greatest interest because experiments and astrophysical phenomena typically involve many wave periods. The guiding-center equations provide the tools to carry out the averaging and determine the motion of electrons under the (previously calculated) instantaneous Lorentz-Dirac radiation force [4,5]. We give the restrictions on field strength and frequency for our result to be applicable below.

The second part of this paper considers the destruction of the enhanced-mass behavior and transition to stochasticity as the strength of the background fields is increased to violate the field-strength condition $eB_{\mathrm{back}}/mc\omega_{\mathrm{wave}} \ll 1$ necessary for the enhanced-mass derivation. The breakdown of the enhanced-mass picture is shown numerically to predict the onset of stochasticity even for very strong wave intensity. Both the nonlinear radiation acceleration and the stochasticity are expected to be significant in astrophysical situations, and the first effect may well be visible in laboratory experiments with ultraintense laser pulses.

Classical calculations are valid for high a as long as the discrete photon energy $\hbar\omega \ll mc^2$ and the amplitude for QED processes such as e^+e^- pair creation is small ($eE\hbar/mc \ll mc^2$) [6]. The classical high-a regime includes a number of existing and proposed accelerator designs, such as the plasma wakefield and beat-wave accelerators [7], and nonlinear multiphoton effects such as Compton-like scattering [8]. Lasers used in the National Ignition Facility and similar inertial fusion projects have $a \sim 1$ so strong-field effects are significant. Many astrophysical problems also involve high-a radiation sources, and in particular consequences of strong-wave radiation damping are discussed in [9].

The motion of an electron in a plane wave of arbitrary intensity is integrable both classically and within the Dirac equation. This occurs because a third conserved quantity $mc\gamma - p_x$ exists in addition to the two transverse generalized momenta. Here γ is the electron Lorentz factor and p_x is the component of electron momentum along the wave axis. Including radiative effects or adding additional fields generally violates this conservation. The result of the scattering of the wave by the electron is a force on the electron along the wave axis, which has the form $F = 2e^4\langle \mathbf{E}^2\rangle/3m^2c^4$ for weak plane waves. In the high-a regime the electron radiates much more strongly ($\propto a^4$ rather than a^2 in the low-a limit) and radiates high harmonics of the wave frequency [4,5]. Some care must be taken to calculate the radiation acceleration for a strong plane wave correctly, since the electron can have different γ and hence different inertia at different times in its motion in the plane wave. Another way to understand this is that the conservation of momentum along the wave axis, which is sufficient to give the time-averaged acceleration for a weak wave (equal and opposite the averaged radiation force), does not fix the acceleration for a strong wave because the electron momentum along the wave axis in a strong wave is not constant. It will be seen that naive conservation of momentum is satisfied if a drifting electron is attributed average momentum $m\gamma_0 v_d$, where $m\gamma_0$ is the enhanced mass and v_d is the drift velocity, assumed nonrelativistic.

Previous work on the acceleration due to radiation reaction in a strong wave has included numerical studies [10,6] as well as rather complicated exact solutions to the Lorentz-Dirac equation (in the Landau approximation [4]) in special cases, such as monochromatic linearly and circularly polarized waves [11]. For an extremely strong wave there are well-known difficulties in treating the electron motion classically, due to the existence of acausal ''runaway'' solutions to the Lorentz-Dirac equation. The Landau approximation, which is valid when the radiation reaction force is much smaller than the other forces on the electron (as is the case for all currently available laser field strengths), consists of calculating the radiation force from the instantaneous acceleration in the absence of the radiation force. The work discussed here is independent of the various attempts to find a consistent classical description of strong radiation reaction forces (see references in [6]) when the Landau approximation is not valid.

For laser fields in the laboratory and in all but the most exotic astrophysical situations, radiation reaction can be treated as a perturbation and the Landau approximation is valid. In this regime, the guiding-center formalism gives a simple result for the drift velocity, the quantity of primary interest, valid for an arbitrarily polarized, polychromatic wave, even though the electron motion is highly relativistic. The two approximations involved are the Landau approximation and the guiding-center approximation that the changes in the drift velocity over a single wave period are nonrelativistic and hence add linearly, which is very well satisfied for the radiation reaction problem in the classical regime.

The motion in a wave along $\hat{\mathbf{x}}$ with dimensionless vector potential $\mathbf{A}(t-x/c)=\mathbf{A}(\eta)$ is given in the ''drift frame'' where the electron has zero average velocity by $p_i=-mcA_i$ for the transverse components, and $p_x=mc(\mathbf{A}^2-\langle\mathbf{A}^2\rangle)/2\gamma_0$, with $\langle\rangle$ indicating averaging over η and $\gamma_0^2=1+\langle\mathbf{A}^2\rangle$. There are several equivalent expressions for γ_0 for the electron trajectory in the wave with no external or radiative forces: γ_0 is equal to the constant of motion $\gamma-p_x/mc$ in the frame where the electron is at rest on average. Hence $d\eta/dt=1-v_x/c=\gamma_0/\gamma$ in this frame, and the mass enhancement factor works out to γ_0:

$$\frac{1}{\langle 1/\gamma\rangle_{\text{time}}}=\frac{1}{\langle 1/\gamma_0\rangle_{\text{phase}}}=\gamma_0. \tag{1}$$

The wave envelope is assumed constant in η so that the averages are well-defined. Variation in the wave envelope causes a ponderomotive force, reviewed below.

The radiation reaction force from scattered photons causes the drift velocity (the velocity of the drift frame) to change in time. The change in drift velocity for an applied impulse requires some calculation [3] which we will not repeat here: the idea is to consider the infinitesimal Lorentz transformation required to restore the electron to its usual point on the wave trajectory. It turns out that unexpected terms arise in the equation for the drift velocity, and that consequently the instantaneous drift acceleration from an applied force is not necessarily in the direction of the applied force. For the radiation reaction problem in a weak wave, it is these terms which give the usual radiation force along the wave axis, even though the instantaneous radiation force is always perpendicular to the wave. The drift velocity component along the wave axis changes for an applied force $\mathbf{F}$ (in addition to the force from the wave) according to [3]

$$\frac{dv_d^x}{dt}=\frac{F_x}{m\gamma}-\frac{F_y v_y}{mc\gamma_0}-\frac{F_z v_z}{mc\gamma_0}. \tag{2}$$

The causality problems with the Lorentz-Dirac classical radiation force [6] do not appear in the Landau approximation, which is valid if the radiation force is a small perturbation on the motion causing the radiation. The instantaneous radiation force on the electron is [4,5]

$$\begin{aligned}F_i&=\frac{2e^2}{3mc^3}\left\{\frac{d}{dt}\left(\gamma\frac{dp_i}{dt}\right)-\frac{v_i\gamma^2}{mc^2}\left[\left(\frac{d\mathbf{p}}{dt}\right)^2-\left(\frac{dE}{cdt}\right)^2\right]\right\}\\&=\frac{2e^2\gamma_0^2}{3mc^3}\left\{\frac{d^2p_i}{d\eta^2}-\frac{v_i\gamma_0^2}{mc^2}\left[\left(\frac{d\mathbf{p}}{d\eta}\right)^2-\left(\frac{dE}{cd\eta}\right)^2\right]\right\}.\end{aligned} \tag{3}$$

Upon substituting Eq. (3) into Eq. (2), we obtain the instantaneous acceleration (with respect to phase) in the x direction,

$$\frac{dv_d^x}{d\eta}=\frac{2e^2\gamma_0}{3mc^2}\left(\frac{d\mathbf{A}}{d\eta}\right)^2. \tag{4}$$

Now the averaging over η is simple. For a wave with intensity per frequency interval $I(\omega)$, the result is

$$\left\langle\frac{dv^x}{dt}\right\rangle=\frac{8\pi e^4\int I(\omega)\,d\omega}{3m^3c^5}\sqrt{1+\frac{4\pi e^2}{m^2c^3}\int\frac{I(\omega)\,d\omega}{\omega^2}}. \tag{5}$$

The acceleration is thus independent of polarization and the phasing between different frequences, just as in the nonrelativistic limit, but scales with a^3 rather than a^2 for large a with ω constant, since then $I(\omega)\propto a^2$.

For a linearly polarized monochromatic wave, the average acceleration is $2e^2\omega^2(a^2/2+a^4/4)\sqrt{1+a^2/2}/3mc^2$. For circular polarization, the result is $2e^2\omega^2(a^2+a^4)\sqrt{1+a^2}/3mc^2$. (In these expressions $a=eE_{\max}/mc\omega$ for both polarizations.) For circular polarization this is just the radiated power divided by $\bar{m}c$, with $\bar{m}=m\gamma_0$. For linear polarization this is not the case, but the correct acceleration is obtained if we account for the fact that the electron's total radiated momentum is nonzero, unlike for circular polarization [5]. That is, the total transfer of momentum from the wave along the wave axis $\mathbf{k}$ is $P_{\text{rad}}/c+M_{\mathbf{k}}$, where P_{rad} is the total radiated power and $M_{\mathbf{k}}$ is the rate of $\mathbf{k}$ momentum radiated. The correct average radiation acceleration is thus obtained from conservation of energy and momentum *if* a drifting electron is assigned momentum $m\gamma_0 v_d$. However, there is no *a priori* reason for this assignment without the use of Eq. (2).

The average radiation force can be defined as the required constant force to balance the above acceleration: $F_{\mathrm{rad}} = m\gamma_0 A_{\mathrm{rad}}$. The radiation force from a blackbody source of intensity I and temperature T is

$$F_{\mathrm{rad}} = \frac{8\pi e^4 I}{3m^2c^5}\left(1+\frac{10\hbar^2 e^2 I}{\pi m^2 c^3 k_B^2 T^2}\right). \tag{6}$$

It may be useful to clarify the differences between the radiation force calculated above and the longitudinal ponderomotive force. Changes in the envelope of a plane wave cause a ponderomotive force, which conserves $mc\gamma - p_x$ and is directed forward when the wave is rising and backward when the wave is falling. That this force conserves $mc\gamma - p_x$ can be seen from the exact solution for motion in a plane wave neglecting radiation [4]. (There are also transverse ponderomotive forces in real beams arising from the finite spot size; these have a different character and are not discussed here. Transverse ponderomotive forces are important in a number of real-world situations; the reason they are not discussed here is that a transverse ponderomotive force occurs when the wave is not a plane wave, but the guiding-center picture is only rigorously correct for a plane wave. It seems likely that as long as the percentage falloff $\Delta a/a$ over a wavelength in the transverse direction is much less than unity, the guiding-center picture with a transverse ponderomotive force will be a good description, but this can probably only be demonstrated by numerical calculation, unlike the case of the longitudinal force [3].) The radiation force does not conserve $mc\gamma - p_x$ and always acts in the forward direction. Treating an electron in a plane wave as an enhanced-mass particle acted upon by ponderomotive and radiation forces gives a simple and accurate description of single-particle behavior in the classical regime.

The reader may wonder why the radiation field causing the above force was not studied in detail. The instantaneous force on the electron is given in the Landau approximation by Eq. (3), which is equal and opposite to the rate of momentum carried away calculated by Sarachik and Schappert [5]. The change in motion from the radiation force then causes changes in the radiated fields, which in turn cause further changes in the electron motion, and so on and so forth. The idea of the Landau approximation is that this expansion around the original motion and fields can be truncated if the radiation force is much smaller than the original force producing the radiation. In essence, there are indeed changes in the radiated fields due to the motion in the radiation force, but these are sufficiently small that it is appropriate to take the radiated fields as the fields of [5], centered on the electron position. Constructing a classical electron model which is consistent even when the radiation force is as strong as the original force is a deep challenge. However, the physical appropriateness of the Landau approximation for relatively weak radiation forces is shown by the correct results obtained for radiation damping in accelerators and many other problems.

Above we focused on the radiation force on a single particle in a strong wave; we now digress to consider the effects of this force on a plasma of charged particles. In laboratory experiments, the pulse length is sufficiently short that it is often appropriate to consider the electrons as independent for the length of the pulse, and then to consider collective effects resulting from the modification of the particle distribution in phase space by the pulse. In astrophysical situations, however, and for some current and future experiments, it is necessary to consider how the collective behavior of particles is changed while they are in the wave. This is much more difficult than the single-particle behavior would lead one to believe. The radiation force on a single particle moving in a wave will be unaltered as long as the other forces acting on the particle are much weaker than the wave, which is true for the Coulomb forces from other particles at reasonable densities and also for the radiation fields from other particles oscillating in the wave. Fundamental changes in the collective behavior of the plasma are expected because, for a strong wave, the radiation field produced by a particle, which falls off as r^{-1}, dominates the r^{-2} Coulomb field at a rather short distance, and this radiation field oscillates on the fast time scale of the wave. The behavior of a plasma of particles in a strong wave interacting through radiation in addition to Coulomb fields is a formidable problem, which is likely to exhibit new types of screening and collective modes.

Now we turn to consider the second problem mentioned in the opening: the destruction of the enhanced-mass picture when strong constant electromagnetic fields are added to the plane wave. It will be shown that for one typical field configuration (i.e., one without special symmetries giving rise to integrability), the breakdown of the enhanced-mass picture is consistent with the analytical predictions of [3] and associated with the onset of stochasticity over a wide range of beam intensity. An example of an integrable configuration is an applied magnetic field parallel to the wave axis [12]. The value of the enhanced-mass picture is that, even when the motion cannot be solved exactly (as shown below by numerical computation of Liapunov exponents), the enhanced-mass picture predicts the motion in the quasiperiodic regime even for $a>1$ as well as the boundary between quasiperiodic behavior and chaotic behavior. In the following the wave will be taken to have constant amplitude and the radiation force will be neglected. For definiteness consider adding a constant magnetic field $\mathbf{B}=B\hat{\mathbf{z}}$ to a linearly polarized wave $A_y(\xi)$ traveling in the $\hat{\mathbf{x}}$ direction. Then p_z is constant and we take $p_z=0$ so that the electron motion is confined to the xy plane. For small wave strength $a_w = eE/mc\omega$ the motion can be analyzed perturbatively because the equations of motion are nearly linear [13].

When a_w is of order unity, the equations are strongly nonlinear and new phenomena appear. However, the motion is still simple for $a_w>1$ as long as the applied magnetic field is not too strong. In the derivation of the equations for the motion of the guiding center, it was necessary to assume $a_b = eB/mc\omega = \omega_c/\omega \ll 1$, i.e., the electron is far from resonance. Figure 1 shows a typical trajectory when the enhanced-mass picture is applicable: the electron executes fast oscillations in the wave, while its guiding center makes a

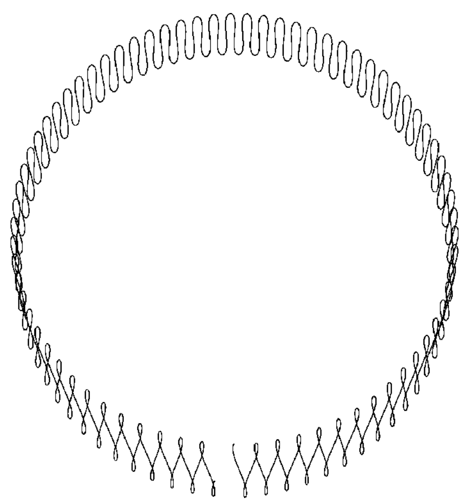

FIG. 1. Typical electron cyclotron motion with $a_w = 1.5$, $a_b = 0.02$, $\mathbf{B} \| \hat{\mathbf{z}}$, and $\mathbf{k} \| \hat{\mathbf{x}}$.

slow orbit in the magnetic field. The relativistic nonlinearity in the equations of motion will be shown to destroy integrability, but only for $\omega_c/\omega \sim 1$.

With no wave present, the gyrocenter of the electron motion $(x_0, y_0) = (x + p_y/eB, y - p_x/eB)$ (with $m = c = 1$) is constant in time. The electron still has a well-defined gyrocenter when the wave is added, which reduces to the normal gyrocenter when the wave strength is zero and is exactly constant for an arbitrarily strong wave. Its coordinates are

$$(K_x, K_y) = \left(x + \frac{p_y + eA_y(\xi)}{eB}, y - \frac{p_x + 1 - \gamma}{eB} \right). \tag{7}$$

Such a gyrocenter exists for any direction of the magnetic field $B_0 \hat{\mathbf{b}}$ and any polarization of the wave: $\mathbf{r}_c = \mathbf{r} - (\mathbf{r} \cdot \hat{\mathbf{b}}) \hat{\mathbf{b}} - \hat{\mathbf{b}} \times [\mathbf{p} + e\mathbf{A}(\xi) + 1 - \gamma \hat{\mathbf{k}}]/eB_0$.

Now consider again the particular case $\hat{\mathbf{k}} \| \hat{\mathbf{x}}, \mathbf{A} \| \hat{\mathbf{y}}, \mathbf{B} \| \hat{\mathbf{z}}$. Taking $p_z = 0$ confines the motion to the xy plane so that phase space is five-dimensional [position (x,y), momentum (p_x, p_y), and time t]. The constants (7) reduce the effective dimension of phase space by 2. Choosing particular values $K_x = K_y = 0$ of the constants corresponds to shifting the gyrocenters of all possible trajectories to the origin, removing two translational degrees of freedom. After this shift, p_x and p_y are no longer independent coordinates but rather functions of x and y determined by Eq. (7).

The existence of two constants of motion reduces the effective phase space in this particular case from five dimensions to three. Hamiltonian motion in a two-dimensional phase space is always integrable, so that three-dimensional motions such as the driven pendulum and the Chirikov-Taylor problem [14] are the simplest that can exhibit nonintegrable behavior. The equations of motion after a change to the independent variable $\eta = t - x/c$ and introducing dimensionless parameters $a_b = \omega_c/\omega, a_w = eE_0/mc\omega$, and a monochromatic wave $A(\eta)$ are $(\omega = m = c = 1)$

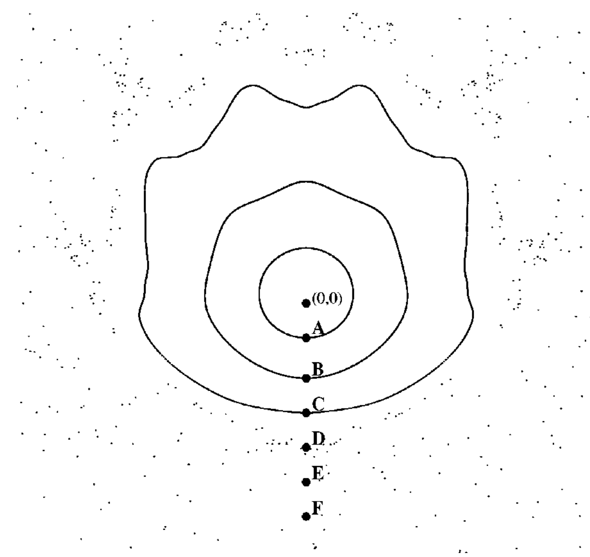

FIG. 2. Surface of section showing trajectories from six different initial conditions with $a_w = 0.1$, $a_b = 0.18$.

$$\gamma = 1 + \frac{a_b^2 y^2 + (a_b x - a_w \sin \eta)^2}{2(1 + a_b y)},$$
$$\frac{dx}{d\eta} = \frac{\gamma}{1 + a_b y} - 1, \quad \frac{dy}{d\eta} = \frac{a_b x - a_w \sin \eta}{1 + a_b y}. \tag{8}$$

These equations are integrated numerically for various values of a_b, a_w, and the initial conditions $x(\eta_0), y(\eta_0)$.

Because the equations of motion (8) are periodic in η, it is convenient to plot the electron's (x,y) coordinates after each period to study the long-time behavior. This gives an area-preserving map of the plane to itself. Figure 2 shows a typical surface of section obtained in this way. For low values of the electron initial energy, the motion is quasiperiodic and nearly circular, while for large values of the initial energy the motion is chaotic, as demonstrated by numerically calculated Liapunov exponents (Fig. 3). As time increases, the largest exponent for trajectories beginning on points D, E, F remains positive, indicating that neighboring initial points separate exponentially rapidly [15]. The electron's initial energy affects the character of the motion because a_b can be much larger in the rest frame of an electron than in the lab

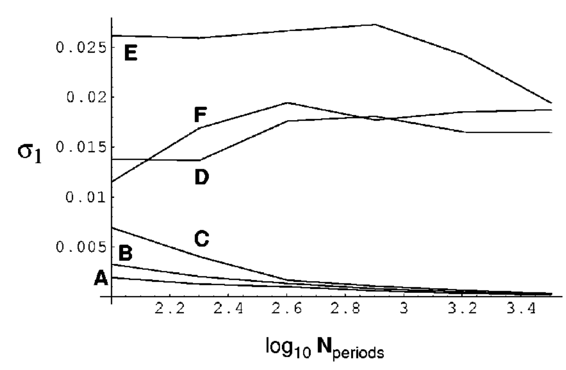

FIG. 3. Numerical largest Liapunov exponent σ_1 calculated at different times for trajectories starting on the six labeled points in Fig. 2.

frame if the electron has a large initial velocity. To eliminate this effect, the initial electron drift velocity is fixed at $0.5c$ henceforth.

The enhanced-mass description predicts that the guiding center of the electron moves in a circle: $(x_{\mathrm{gc}}(t),y_{\mathrm{gc}}(t)) = (x_0+(v_d/\Omega)\sin\Omega t, y_0+(v_d/\Omega)\cos\Omega t)$, with $v_d=0.5c$ and $\Omega=\omega_c/\gamma_0\gamma_d=a_b\omega\sqrt{1-v_d^2}/\gamma_0$. At the end of each wave period, $x_{\mathrm{gc}}(t)$ and $y_{\mathrm{gc}}(t)$ are compared with the actual location of the guiding center calculated from Eq. (8). As a dimensionless measure of the error, we use the normalized sum of squares error over a cyclotron orbit:

$$E_{\mathrm{gc}}=\left(\frac{\Omega}{v_d}\right)^2 \sum_{\omega(t_n-x_n/c)=2n\pi}^{\Omega t_n<2\pi} [\mathbf{r}(t)-\mathbf{r}_{\mathrm{gc}}(t)]^2. \tag{9}$$

The error is found to be quite small ($E_{\mathrm{gc}}<0.01$) for all values of a_w studied as long as $a_b<0.04$. For each value of a_w, E_{gc} increases rapidly to order unity once a_b reaches a certain critical field: as a threshold we define $a_b^{\mathrm{crit}}(a_w)$ as the value of a_b where $E_{\mathrm{gc}}=0.01$. Consistent with the predictions of the enhanced-mass picture, a_b^{crit} remains nonzero for large a_w (and in fact increases slightly with a_w). Even though high a_w makes Eqs. (8) quite nonlinear, the motion remains quasiperiodic and nearly circular for $a_b<a_b^{\mathrm{crit}}$.

As a_b increases above a_b^{crit}, the trajectory with initial velocity $0.5c$ becomes chaotic (has a positive Liapunov exponent) at some value a_b^*. For large a_w, above a_b^* the motion is strongly chaotic and the electron energy fluctuates wildly. This differs from nearly linear resonance at small a_w in that no tuning of frequencies is necessary for energy gain and hence energy gain is not limited by relativistic detuning. Figure 4 shows numerical curves for a_b^{crit} and a_b^* as part of a schematic phase diagram. The two curves are adjacent over four decades of beam intensity. The dotted line (which was not calculated and is only schematic) separating the quasilinear resonance phase from the strong stochasticity phase can be defined by the destruction of the last invariant torus at large energy, since in three dimensions such a torus bounds the energy of trajectories contained within it. Rax has previously proposed that the stochastic motion of electrons in *multiple* plane waves may give rise to high-energy cosmic rays [16]. The considerations above indicate that a single plane wave, together with a sufficiently strong magnetic field, is sufficient.

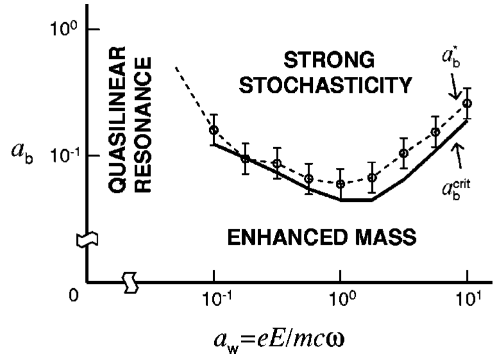

FIG. 4. Numerical values a_b^{crit} and a_b^* for various a_w as part of a schematic phase diagram. Error bars are shown for a_b^* because it is difficult to determine precisely when the largest Liapunov exponent becomes positive.

We verified that approximately the same boundary for the guiding-center description applies when the applied wave is a superposition of two or three applied frequencies. It seems natural to conjecture that the guiding-center region in Fig. 4 also describes waves of finite bandwidth and other orientations of the magnetic field (excluding the integrable case $\mathbf{B}\|\mathbf{k}$).

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